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Clebsch-Gordan Coefficients in Rational Form

Function subprogram `RTCLGN` calculates the (signed) square of the Clebsch-Gordan coefficient in rational form and in powers of prime numbers. In terms of the Wigner-$3j$ symbol, this coefficient is defined by

$$C \;=\; (j_1\, j_2\, m_1\, m_2 \,|\, j_1\, j_2\, j_3\, m_3) \;=\; (-1)^{j_1-j_2+m_3}\,\sqrt{2j_3+1}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix}.$$

All j_i and m_i must have integral or half-integral values (see **Notes**). For definitions and notations see Ref. 1.

On computers other than CDC and Cray, only the double-precision version `DTCLGN` is available. On CDC and Cray computers, only the single-precision version `RTCLGN` is available.

Structure:

`SUBROUTINE` subprogram
User Entry Names: `RTCLGN`
Files Referenced: `Unit 6`

Usage:

For `t` = `R` (type `REAL`), `t` = `D` (type `DOUBLE PRECISION`),

```
CALL tTCLGN(JJ1,JJ2,JJ3,MM1,MM2,MM3,RNUM,RDEN,KPEX)
```

`JJ1,JJ2,JJ3` (`INTEGER`) The j-parameters multiplied by two, i.e. `JJ1`$= 2j_1$ etc.

`MM1,MM2,MM3` (`INTEGER`) The m-parameters multiplied by two, i.e. `MM1`$= 2m_1$ etc.

`RNUM` (type according to `t`) Contains, on exit, the signed numerator of C^2.

`RDEN` (type according to `t`) Contains, on exit, the denominator of C^2.

`KPEX` (`INTEGER`) Array of length 40 at least. Contains, on exit, the exponents k_n in the expression

$$C^2 \;=\; \prod_{n=1}^{40} p_n^{k_n},$$

where $p_1 = 2,\ p_2 = 3,\ p_3 = 5, \ldots,\ p_{40} = 173$ are the first 40 prime numbers.

Notes:

A Clebsch-Gordan coefficient $(j_1\, j_2\, m_1\, m_2 \,|\, j_1\, j_2\, j_3\, m_3)$ is considered to be zero unless simultaneously

(i) j_i and m_i have both either integral or half-integral values (each i),
(ii) $j_i \geq |m_i| \geq 0$ (each i),
(iii) $m_1 + m_2 = m_3$,
(iv) $j_1 + j_2 + j_3$ is an integer and $j_1 + j_2 \geq j_3,\quad j_2 + j_3 \geq j_1,\quad j_3 + j_1 \geq j_2$.

In this case, `RNUM` = `0`, `RDEN` = `1` or `DNUM` = `0`, `DDEN` = `1`, respectively, and `KPEX(n)` = `0`, (`n` = `1`, ... , `40`).

Source:

This subroutine is based on an earlier version by H. Yoshiki.

Error handling:

Error `U112.1`: The calculation requires a prime number p_n with $n > 40$.
In this case, $\mathtt{DNUM} = 0$, $\mathtt{DDEN} = 1$, $\mathtt{KPEX(n)} = 0$, $(\mathtt{n} = 1, \ldots, 40)$. A message is written on `Unit 6` unless subroutine `MTLSET` (N002) has been called.

References:

1. R.D. Cowan, The theory of atomic structure and spectra, (Univ. of California Press, Berkeley CA 1981) 142–144

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