Author(s) : W. Hart
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Manipulation of Triangular and Symmetric Matrices

At CERN, matrices are often stored row-wise (TC-convention); furthermore, symmetric matrices are stored packed as the lower left triangular part only, i.e., the Ith diagonal element is found in position $I(I+1)/2$. The TR-package performs many of the frequently required operations associated with such matrices without resorting to expanding into the unpacked square form. In all the following routines an $M \times M$ *symmetric* matrix is taken to be stored in the packed form with $M(M+1)/2$ elements.

Some of these operations produce and require the manipulation of *lower triangular* matrices which have all elements zero above the leading diagonal. These are also stored in the packed form with all the zeros dropped; therefore, care has to be taken in the interpretation of a packed matrix as to whether it represents a symmetric or lower triangular array. To facilitate this distinction in the Write-up, the following nomenclature has been adopted:

A,B,C	unpacked rectangular matrices (row-wise storage)
Q,R,S,T	packed symmetric matrices
V,W	packed lower triangular matrices

On 32-bit machines the calculations are performed internally in double-precision mode.

Structure:

SUBROUTINE subprograms
User Entry Names: TRCHUL, TRCHLU, TRSMUL, TRSMLU, TRINV, TRSINV, TRLA, TRLTA, TRAL, TRALT, TRSA, TRAS, TRSAT, TRATS, TRAAT, TRATA, TRASAT, TRATSA, TRQSQ, TRPCK, TRUPCK

Usage:

Choleski Decomposition

```
CALL TRCHUL(S,W,M)
CALL TRCHLU(S,V,M)
```

$\mathbf{S} = \mathbf{W}'\mathbf{W}$

$\mathbf{S} = \mathbf{V}\mathbf{V}'$

S is an $M \times M$ *positive semi-definite* symmetric matrix (e.g., error or weight matrix) and the routines calculate the complementary lower triangular Choleski factors. It is allowed to overwrite S by W or V.

Symmetric Multiplication of Lower Triangular Matrices

```
CALL TRSMUL(W,S,M)
CALL TRSMLU(W,R,M)
```

$\mathbf{W}'\mathbf{W} \to \mathbf{S}$

$\mathbf{W}\mathbf{W}' \to \mathbf{R}$

W is an $M \times M$ lower triangular matrix and S, R the two symmetric products of the multiplication of W by its transpose. It is allowed to overwrite W by either S or R.

Lower Triangular Matrix Inversion

```
CALL TRINV(W,V,M)
```
$\mathbf{W}^{-1} \to \mathbf{V}$

`W` is an `M` $\times$ `M` lower triangular matrix which is inverted into `V` (the inverse of a lower triangular matrix is lower triangular). `W` *may have rows and columns of zeros* as produced by the Choleski decomposition of a weight matrix with unmeasured variables. It is allowed to overwrite `W` by `V`.

Symmetric Matrix Inversion

```
CALL TRSINV(S,R,M)
```
$\mathbf{S}^{-1} \to \mathbf{R}$

`S` is an `M` $\times$ `M` *positive semi-definite* symmetric matrix which is inverted into `R` (also stored packed). It is permissible to overwrite `S` by `R`.

Triangular – Rectangular Multiplication

`CALL TRLA (W,A,B,M,N)`	$\mathbf{WA} \to \mathbf{B}$
`CALL TRLTA(W,A,B,M,N)`	$\mathbf{W'A} \to \mathbf{B}$
`CALL TRAL (A,V,B,M,N)`	$\mathbf{AV} \to \mathbf{B}$
`CALL TRALT(A,V,B,M,N)`	$\mathbf{AV'} \to \mathbf{B}$

`A` and `B` are `M` $\times$ `N` rectangular matrices, `W` is an `M` $\times$ `M` lower triangular matrix, and `V` is an `N` $\times$ `N` lower triangular matrix. In each call it is allowed to overwrite `A` by `B`.

Symmetric - Rectangular Multiplication

`CALL TRSA (S,A,C,M,N)`	$\mathbf{SA} \to \mathbf{C}$
`CALL TRAS (A,R,C,M,N)`	$\mathbf{AR} \to \mathbf{C}$
`CALL TRSAT(S,B,C,M,N)`	$\mathbf{SB'} \to \mathbf{C}$
`CALL TRATS(B,R,C,M,N)`	$\mathbf{B'R} \to \mathbf{C}$

`A` and `C` are `M` $\times$ `N` rectangular matrices, `B` is an `N` $\times$ `M` matrix, `S` is an `M` $\times$ `M` symmetrix matrix, and `R` is an `N` $\times$ `N` symmetric matrix. It is *not* allowed to overwrite `A` or `B` by the product matrix `C`.

Symmetric Multiplication of Rectangular Matrices

`CALL TRAAT(A,S,M,N)`	$\mathbf{AA'} \to \mathbf{S}$
`CALL TRATA(B,R,M,N)`	$\mathbf{B'B} \to \mathbf{R}$

`A` is an `M` $\times$ `N` matrix, `B` is an `N` $\times$ `M` matrix, `S` is an `M` $\times$ `M` symmetric matrix, and `R` is an `M` $\times$ `M` symmetric matrix. No overwriting is allowed.

Transformation of Symmetric Matrix

`CALL TRASAT(A,S,R,M,N)`	$\mathbf{ASA'} \to \mathbf{R}$
`CALL TRATSA(B,S,R,M,N)`	$\mathbf{B'SB} \to \mathbf{R}$
`CALL TRQSQ (Q,T,R,M)`	$\mathbf{QTQ} \to \mathbf{R}$

`A` is an `M` $\times$ `N` matrix, `B` is an `N` $\times$ `M` matrix, `S` is an `N` $\times$ `N` symmetric matrix, and `R`, `Q`, `T` are `M` $\times$ `M` symmetric matrices. No overwriting is allowed.

Packing and Unpacking a Symmetric Matrix

`CALL TRPCK (A,S,M)`	$\mathbf{A} \to \mathbf{S}$
`CALL TRUPCK(S,A,M)`	$\mathbf{S} \to \mathbf{A}$

`A` is an `M` $\times$ `M` unpacked symmetric matrix (all $\mathtt{M}^2$ elements) and `S` is the same matrix stored packed. Overwriting is allowed for both `TRPCK` and `TRUPCK`.

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