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Cubic Splines and their Integrals

Subroutines RCSPLN and DCSPLN compute a (vector-valued) cubic spline function $F(x)$ which interpolates between a given set of points. Entries RCSPNT and DCSPNT compute the first and second integral over $F(x)$.

On computers other than CDC or Cray, only the double-precision versions DCSPLN and DCSPNT are available. On CDC and Cray computers, only the single-precision versions RCSPLN and RCSPNT are available.

Given an interval $[a, b]$, a subdivision of this interval into $n \geq 2$ subintervals

$$a = x_0 < x_1 < ... < x_{n-1} < x_n = b,$$

and $n+1$ function values $Y_k = \{y_{k1}, \ldots, y_{km}\}$ on the $n+1$ abscissae (called 'knots') x_k $(k = 0, 1, \ldots, n)$, RCSPLN and DCSPLN compute a function $F(x)$ of class C^2, defined on $[a, b]$, which assumes the given value Y_k at the knot x_k (i.e. $F(x_k) = Y_k$), and which, when restricted to the ith sub-interval $[x_{i-1}, x_i]$ is identical with a set of m polynomials $\{p_{i1}, ..., p_{im}\}$, each of degree at most 3. Any function $F(x)$ which satisfies the above two conditions is called a 'cubic spline' through the $n + 1$ points (x_k, Y_k). To define the spline function $F(x)$ uniquely the subroutines impose an additional boundary condition, specified by their MODE parameter:

MODE = 0: $F''(x_0) = F''(x_n) = 0$ (the so-called *natural spline*).

MODE = 1: $F''(x_0) = F''(x_1)$ and $F''(x_{n-1}) = F''(x_n)$.

Structure:

SUBROUTINE subprograms
User Entry Names: RCSPLN, RCSPNT, DCSPLN, DCSPNT
Files referenced: Unit 6

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

```
Spline:     CALL tCSPLN(N,X,M,Y,NDIM,MODE,A,B,C,D)
Integrals:  CALL tCSPNT(N,X,M,Y,NDIM,MODE,A,B,C,D)
```

N (INTEGER) Number n of subintervals $[x_{i-1}, x_i]$. Must contain a value of at least 2 on entry. Unchanged on return.

X (type according to t) One-dimensional array of dimension (0:d) of at least $n+1$ elements. On entry, X(k) must contain the abscissa x_k, $(k = 0, 1, \ldots, n)$. Unchanged on return.

M (INTEGER) Number m of components of the vector-valued spline function $F(x)$. Must contain a value of at least 1 on entry. Unchanged on return.

Y (type according to t) Two-dimensional array of dimension (0:NDIM,d) where d is a value not less than m. On entry, Y(k,j) must contain the value y_{kj} of the jth component of the vector Y_k, $(k = 0, 1, \ldots, n;\ j = 1, \ldots, m)$. Unchanged on return.

NDIM (INTEGER) Upper bound of the first dimension of arrays A, B, C, D and Y. Must contain a value of at least n on entry. Unchanged on return.

MODE (INTEGER) Type of boundary condition (see description above). Must contain the value 0 or 1 on entry. Unchanged on return.

`A,B,C,D` (type according to `t`) Two-dimensional arrays of dimension `(NDIM,d)`, where `d` $\geq m$.
On return from `RCSPLN`, `A(i,j)`, `B(i,j)`, `C(i,j)` and `D(i,j)` will contain the four coefficients a_{ij}, b_{ij}, c_{ij}, and d_{ij} of the polynomial

$$p_{ij} = a_{ij} + b_{ij}(x - x_{i-1}) + c_{ij}(x - x_{i-1})^2 + d_{ij}(x - x_{i-1})^3$$

that determines the jth component $f_j(x)$ of the spline in the ith subinterval $[x_{i-1}, x_i]$, $i = l, \ldots, n$, $j = 1, \ldots, m$.
On return from `RCSPNT`,

$$\texttt{A(i,j) = } \int_a^{x_i} f_j(t)\,dt \quad \text{and} \quad \texttt{B(i,j) = } \int_a^{x_i}\int_a^{x} f_j(t)\,dt\,dx,$$

with $i = 1, \ldots, n$; $j = 1, \ldots, m$.
Arrays `C` and `D` have been used as working space.

Restrictions:

`N` $\geq$ `2`, `M` $\geq$ `1`, `NDIM` $\geq$ `N`, `MODE` $=$ `0` or `1`.

Error handling:

Error `E211.1`: `N` $<$ `2`.
Error `E211.2`: `M` $<$ `1`.
Error `E211.3`: `NDIM` $<$ `N`.
Error `E211.4`: `MODE` $\neq$ `0` and `MODE` $\neq$ `1`.
A message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.
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