**Author(s) :** G.A. Erskine and K.S. Kölbig **Library:** MATHLIB
**Submitter :** **Submitted:** 07.06.1992
**Language :** Fortran **Revised:**

## Solution of a Linear Fredholm Integral Equation of Second Kind

Subroutine subprograms RFRDH1, DFRDH1 and function subprograms RFRDH2, DFRDH2 calculate an approximation to the solution $y$ of the Fredholm integral equation

$$y(x) \;=\; F(x) + \int_a^b G(x,t)\,y(t)\,dt \qquad (1)$$

over the interval $[a,b]$. The function $F$ must not be identically zero. The interval $[a,b]$ may be divided into $m$ subintervals $[t_{i-1},t_i]$, $(i=1,2,\ldots,m)$, with $a=t_0<t_1<\cdots<t_m=b$.
The order $N_i$ (number of abscissae) of the Gaussian quadrature formula used for integrating over $[t_{i-1},t_i]$ is specified separately for each subinterval.

Function subprograms RFRDH3 and DFRDH3 evaluate numerically integrals of the form $\int_a^b H(t)\,y(t)\,dt$ where $H$ is an arbitrary function and $y$ is the solution of (1).
The following values of $N_i$ may be used: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 20, 24, 32, 40, 48, 64, 80, 96.
On computers other than CDC and Cray, only the double-precision versions DFRDH1 etc. are available. On CDC and Cray computers, only the single-precision versions RFRDH1 etc. are available.

**Structure:**

SUBROUTINE and FUNCTION subprograms
User Entry Names: RFRDH1, RFRDH2, RFRDH3, DFRDH1, DFRDH2, DFRDH3
Files Referenced: Unit 6
External References: RGSET (D107), DGSET (D107), REQN (F010), DEQN (F010), MTLMTR (N002), ABEND (Z035), user-supplied FUNCTION subprograms.

**Usage:**

For t = R (type REAL), t = D (type DOUBLE PRECISION), the first step in the solution of (1) must be the execution of a statement of the form:

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CALL tFRDH1(F,G,M,T,NG,WS,IDIM,N)
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F,G (type according to t) Names of user-supplied FUNCTION subprograms, declared EXTERNAL in the calling program. Subprogram F must set F(X) $=F($X$)$, subprogram G must set G(X, T) $=G($X, T$)$.

M (INTEGER) Number $m\geq 1$ of subintervals in $[a,b]$.

T (type according to t) One-dimensional array of dimension (0:d) where d $\geq$ M. On entry, T must contain the $m+1$ ordered points of subdivision $t_i$, $(i=0,1,\ldots,m)$, with $t_0=a$ and $t_m=b$.

NG (INTEGER) One-dimensional array of length $\geq$ M. On entry, NG must contain the order (number of absissae) $N_i$ of the Gaussian quadrature formula to be used in the interval $t_{i-1}\leq t\leq t_i$, $(i=1,2,\ldots,m)$.

WS (type according to t) Two-dimensional array of dimensions (IDIM, $\geq$ IDIM + 4). Used as working space and for communication between the subprograms.

IDIM (INTEGER) A number $\geq \sum_{i=1}^m N_i$.

N (INTEGER) On exit, N $=\sum_{i=1}^m N_i$.

Once tFRDH1 has been called, the function subprograms tFRDH2 and tFRDH3 may be referenced any number of times without any further call to tFRDH1.
In any arithmetic expression,

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tFRDH2(F,G,X,WS,IDIM,N)
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has the value $y(\mathtt{X})$, where $y$ is the approximate solution of (1).
In any arithmetic expression,

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tFRDH3(H,WS,IDIM,N)
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has the approximate value of $\int_a^b H(t)\, y(t)\, dt$ where $y$ is the approximate solution of (1).
H (type according to t) is the name of a user-defined FUNCTION subprogram, declared EXTERNAL in the calling program. This subprogram must set $\mathtt{H(X)} = H(\mathtt{X})$.

**Method:**

Let the sets $\{w_k\}$ and $\{z_k\}$ be defined by

$$\begin{aligned}\{w_k\} &= \{w_1^{(1)}, \ldots, w_{N_1}^{(1)}, \ldots, w_1^{(m)}, \ldots, w_{N_m}^{(m)}\},\\ \{z_k\} &= \{z_1^{(1)}, \ldots, z_{N_1}^{(1)}, \ldots, z_1^{(m)}, \ldots, z_{N_m}^{(m)}\}.\end{aligned}$$

$w_j^{(i)}$ and $z_j^{(i)}$ are respectively the weights and abscissae of the $N_i$-point Gaussian quadrature formulae corresponding to the interval $[t_{i-1}, t_i]$. Subprograms RFRDH1 or DFRDH1 sets up and solves the following system of simultaneous linear equations with unknowns $y(z_k)$:

$$y(z_k) = F(z_k) + \sum_{j=1}^{N} w_j G(z_k, z_j) y(z_k) \qquad (k = 1, 2, \ldots, N)$$

where $N = \sum_{i=1}^{m} N_i$.

Function subprogram tFRDH2 calculates $\qquad y(\mathtt{X}) = \sum_{k=1}^{N} w_k\, G(\mathtt{X}, z_k)\, y(z_k).$

Function subprogram tFRDH3 calculates $\qquad I = \sum_{k=1}^{N} w_k\, H(z_k)\, y(z_k).$

**Accuracy:**

The accuracy depends upon the extend to which the product $G(x,t)y(t)$ can be represented by a polynomial of degree $2N_i - 1$ for all $x$ in the interval $[t_{i-1}, t_i]$, $(i = 1, 2, \ldots, m)$.

**Error handling:**

Error D601.1: In tFRDH1, the system of linear equations is singular. A message is written on Unit 6, unless subroutine MTLSET (N002) has been called.
If any of the values $N_i$ does not appear in the list given above, a message is written on Unit 6 by RGSET or DGSET (D107) unless subroutine MTLSET (N002) has been called.

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