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Minimum of a Function of One Variable

Subroutine subprograms `RMINFC` and `DMINFC` calculate, to a limited specified accuracy, the abscissa of a single local minimum of a real-valued function $f(x)$ lying in a given interval (a,b), together with the function value at the minimum. Although this subprogram may find a minimum under other conditions (see **Notes**), the search interval should contain exactly one local minimum point x with $a < x < b$.

On CDC and Cray computers, the double-precision version `DMINFC` is not available.

Structure:

`SUBROUTINE` subprograms
User Entry Names: `RMINFC`, `DMINFC`
External References: User-supplied `FUNCTION` subprogram

Usage:

For `t` = `R` (type `REAL`), `t` = `D` (type `DOUBLE PRECISION`),

```
CALL tMINFC(F,A,B,EPS,DELTA,X,Y,LLM)
```

`F`	(type according to `t`) Name of a user-supplied `FUNCTION` subprogram, declared `EXTERNAL` in the calling program. This function must set `F(X)` $= f($`X`$)$.
`A,B`	(type according to `t`) On entry, `A` and `B` must specify the end-points a, b of the search interval.
`EPS`	(type according to `t`) On entry, `EPS` must be equal to the accuracy parameter ε (see **Accuracy**).
`DELTA`	(type according to `t`) On entry, `DELTA` must be equal to the parameter δ specifying a tolerance interval near `A` and `B` (see **Accuracy**).
`X`	(type according to `t`) On exit, `X` is the computed approximation to the abscissa of a minimum of the function $f(x)$.
`Y`	(type according to `t`) Contains, on exit, the value of $f($`X`$)$.
`LLM`	(`LOGICAL`) On exit, `LLM` is `.TRUE.` if the relations $\|\mathtt{X}-\mathtt{A}\| > \delta$ and $\|\mathtt{X}-\mathtt{B}\| > \delta$ are both true (i.e. if `X` is the abscissa of a local minimum lying inside the interval $[\mathtt{A},\mathtt{B}]$), and `.FALSE.` otherwise (see **Notes**).

Method:

The so-called *golden section search* is applied (see **References**). This method uses a fixed number n of function evaluations, where $n = [\,2.08 \times \ln\left(|a-b|/\varepsilon\right) + \frac{1}{2}\,] + 1$.

Accuracy:

The accuracy depends on the behaviour of the function and is difficult to measure. For example, a flat minimum results in poor accuracy. This implies that the subprograms are not intended to replace the usual procedures when a minimum of a function is needed in the exact mathematical sense. In any case, a choice of $\varepsilon > 10^{-8}$ in double-precision and of $\varepsilon > 10^{-4}$ in single-precision mode usually results in a relative error of `X` which is smaller than or in the order of ε. A suggested value of δ is $\delta = 10\varepsilon$.

Notes:

1. As a rule, the specified interval (a, b) should contain strictly one local minimum.
2. If this is not the case, and if $f(x)$ is monotonous in (a, b), the subprograms find the minimum at the correct endpoint a or b. `LLM` is set to `.FALSE.` in this case.
3. In all other possible cases, the behaviour of the subprograms is not easy to predict. In particular, in the case of several minimal points inside (a, b), one of them is found, but not necessarily the one with the smallest value of the function.

References:

1. R. Fletcher, Practical methods of optimization (John Wiley & Sons, Chichester 1987) 39–40.
2. W. Krabs, Einführung in die lineare und nichtlineare Optimierung für Ingenieure (BSB B.G. Teubner, Leipzig 1983) 84–86

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