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First-order Differential Equations (Runge–Kutta–Merson)

Subroutine subprograms RDEQMR and DDEQMR advance the solution of the system of $n \geq 1$ simultaneous first-order differential equations

$$\frac{dy_i}{dx} = f_i(x, y_1, \ldots, y_n), \qquad (i = 1, 2, \ldots, n)$$

from a specified value x_1 to a specified value x_2 of the independent variable x. The integration step-length is automatically adjusted to keep the estimated error per step less than a specified value.

On computers other than CDC and Cray, only the double-precision version DDEQBS is available. On CDC and Cray computers, only the single-precision version RDEQBS is available.

Structure:

SUBROUTINE subprograms
User Entry Names : RDEQMR, DDEQMR
Obsolete User Entry Names: DEQMR ≡ RDEQMR
Files Referenced : Unit 6
External References: MTLMTR (N002), ABEND (Z035), User-supplied SUBROUTINE subprogram

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

```
CALL tDEQMR(N,X1,X2,Y,H0,EPS,SUB,W)
```

N	(INTEGER) Number n of equations.
X1	(type according to t) Initial value x_1 of the independent variable x.
X2	(type according to t) Final value x_2 of the independent variable x.
Y	(type according to t) One-dimensional array of length ≥ N. On entry, Y(i), (i = 1, ..., N), must contain $y_i(x_1)$. On exit, Y(i), (i = 1, ..., N), contains approximate values $y_i(x_2)$.
H0	(type according to t) On entry, H0 must contain the proposed initial step-length h_0. On exit, H0 contains the last computed step-length (See also **Method** and **Notes**).
EPS	(type according to t) The requested absolute accuracy ε. (EPS should not be smaller than approximately 10^3 times the machine precision).
SUB	Name of a user-supplied SUBROUTINE subprogram, declared EXTERNAL in the calling program.
W	(type according to t) Array containing at least 6*N elements required as working-space.

The user-supplied subroutine SUB should be of the form

```
SUBROUTINE SUB(X,Y,F)
```

where the variable X and the one-dimensional arrays Y(*) and F(*) are of type t. This subroutine must set

$$\mathtt{F(I)} = f_{\mathtt{I}}(\mathtt{X}, \mathtt{Y(1)}, \ldots, \mathtt{Y(N)}) \qquad (\mathtt{I} = 1, 2, \ldots, \mathtt{N}).$$

Method:

The method is a modification by Merson of the Runge–Kutta method. The initial value of the step-length h is taken to be the first of the numbers $h_0, h_0/2, h_0/4, \ldots$ for which the estimated relative error at the end of the step is less than ε. At each susequent step, an estimate of the integration error for that step (proportional to h^5) is computed. If the corresponding relative error exceeds ε, the current step-length is halfed; if it is less than $\varepsilon/32$ the step-length is doubled. This process is continued until x_2 is reached. (For details, see Ref. 1).

Error handling:

Error `D202.1`: If the requestec accuracy cannot obtained, a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.
For `N` $<$ `1`, or `X1` $=$ `X2` or `H0` $=$ `0`, control is returned to the calling program without any change in `Y`.

Notes:

For well-conditioned systems of equations any reasonable value of the initial step length h_0 may be chosen. For ill-conditioned systems, the initial value of h_0 may be important, and tests with different values are advised. An inappropriate choice may lead to wrong results in such cases.

References:

1. G.N. Lance, Numerical methods for high-speed computers, (Iliffe & Sons, London 1960) 56

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