Author(s) : K.S. Kölbig **Library:** MATHLIB
Submitter : **Submitted:** 15.02.1989
Language : Fortran **Revised:** 01.12.1994

First-order Differential Equations (Gragg–Bulirsch–Stoer)

Subroutine subprograms RDEQBS and DDEQBS advance the solution of the system of $n \geq 1$ simultaneous first-order differential equations

$$\frac{dy_i}{dx} = f_i(x, y_1, \dots, y_n), \qquad (i = 1, 2, \dots, n),$$

from a specified value x_1 to a specified value x_2 of the independent variable x.

On computers other than CDC and Cray, only the double-precision version DDEQBS is available. On CDC and Cray computers, only the single-precision version RDEQBS is available.

Structure:

SUBROUTINE subprograms
User Entry Names : RDEQBS, DDEQBS
Obsolete User Entry Names: DEQBS $\equiv$ RDEQBS
Files Referenced : Unit 6
External References: MTLMTR (N002), ABEND (Z035), user-supplied SUBROUTINE subprogram

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

```
CALL tDEQBS(N,X1,X2,Y,H0,EPS,SUB,W)
```

N (INTEGER) Number n of equations.

X1 (type according to t) Initial value x_1 of the independent variable x.

X2 (type according to t) Final value x_2 of the independent variable x.

Y (type according to t) One-dimensional array of length $\geq$ N. On entry, Y(i), (i = 1, ..., N), must contain $y_i(x_1)$. On exit, Y(i), (i = 1, ..., N), contains approximate values $y_i(x_2)$.

H0 (type according to t) On entry, H0 must contain the proposed initial step-length h_0. On exit, H0 contains the last computed step-length (See also **Method** and **Notes**).

EPS (type according to t) The requested absolute accuracy ε. (EPS should not be smaller than approximately 10^3 times the machine precision).

SUB Name of a user-supplied SUBROUTINE subprogram, declared EXTERNAL in the calling program.

W (type according to t) Array containing at least 36*N elements required as working-space.

The user-supplied subroutine SUB should be of the form

```
SUBROUTINE SUB(X,Y,F)
```

where the variable X and the one-dimensional arrays Y(*) and F(*) are of type t. This subroutine must set

$$\mathtt{F(I)} = f_{\mathtt{I}}(\mathtt{X}, \mathtt{Y(1)}, \dots, \mathtt{Y(N)}) \qquad (\mathtt{I} = 1, 2, \dots, \mathtt{N}).$$

Method:

For the first integration step, starting at $x = x_1$, the step-length h is chosen to be the smallest of the numbers $h_0,\ h_0/2,\ h_0/4, \ldots$ for which not more than 9 stages of internal extrapolation yield an estimated error less than ε. This procedure is repeated until $x = x_2$ is reached. (For details, see Ref. 1).

Error handling:

Error `D201.1`: If the requestec accuracy cannot obtained, a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.
For `N` $<$ `1`, or `X1` $=$ `X2` or `H0` $=$ `0`, control is returned to the calling program without any change in `Y`.

Notes:

For well-conditioned systems of equations any reasonable value of the initial step length h_0 may be chosen. For ill-conditioned systems, the initial value of h_0 may be important, and tests with different values are advised. An inappropriate choice may lead to wrong results in such cases.

Source:

This subroutines is based on an Algol60 procedure given in Ref. 1. The adaption for integration over a given interval (not only over one step) is due to G. Janin.

References:

1. R. Bulirsch and J. Stoer, Numerical treatment of ordinary differential equations by extrapolation methods, Numer. Math. **8** (1966) 1–13.

•