Author(s) : A.C. Genz, A.A. Malik
Submitter : K.S. Kölbig
Language : Fortran
Library: MATHLIB
Submitted: 15.11.1995
Revised:

Adaptive Quadrature for Multiple Integrals over N-Dimensional Rectangular Regions

Subroutine subprograms `RADMUL` and `DADMUL` compute, to an attempted specified accuracy, the value of the integral

$$I_n = \int_{a_n}^{b_n} \int_{a_{n-1}}^{b_{n-1}} \cdots \int_{a_1}^{b_1} f(x_1, x_2, \ldots, x_n)\, dx_1\, dx_2 \cdots dx_n$$

over an n-dimensional rectangular region, where $a_i,\ b_i,\ (i = 1, 2, \ldots, n)$ are constants.

On computers other than CDC and Cray, only the double-precision version `DADMUL` is available. On CDC and Cray computers, only the single-precision version `RADMUL` is available.

Structure:

`SUBROUTINE` subprograms
User Entry Names : `RADMUL`, `DADMUL`
External References: User-supplied `SUBROUTINE` subprogram

Usage:

For `t` = `R` (type `REAL`), `t` = `D` (type `DOUBLE PRECISION`),

```
CALL tADMUL(F,N,A,B,MINPTS,MAXPTS,EPS,WK,IWK,RESULT,RELERR,NFNEVL,IFAIL)
```

`F` (type according to `t`) Name of a user-supplied `FUNCTION` subprogram, declared `EXTERNAL` in the calling program.

`N` (`INTEGER`) Number n of dimensions ($2 \leq$ `N` ≤ 15).

`A,B` (type according to `t`) One-dimensional arrays of length $\geq$ `N`. On entry, `A(i)` and `B(i)`, (`i` $= 1, \ldots,$ `N`), contain the lower and upper limits of integration, respectively. Note that $a_i,\ b_i$ correspond to x_i.

`MINPTS` (`INTEGER`) Minimum number of function evaluations requested. Must not exceed `MAXPTS`.

`MAXPTS` (`INTEGER`) Maximum number ($\geq$ `2**N` + `2N(N + 1)` + `1`) of function evaluations to be allowed.

`EPS` (type according to `t`) Specified *relative* accuracy.

`WK` (type according to `t`) One-dimensional array of length `IWK`, used as working space.

`IWK` (`INTEGER`) Length ($\geq$ `(2N + 3) * (1 + MAXPTS/(2**N + 2N(N + 1) + 1))/2`) of `WK`.

`RESULT` (type according to `t`) Contains, on exit, an approximate value of the integral I_n.

`RELERR` (type according to `t`) Contains, on exit, an estimation of the relative accuray of `RESULT`.

`NFNEVL` (`INTEGER`) Contains, on exit, the number of function evaluations performed.

IFAIL (INTEGER) On exit:

- 0 Normal exit. RELERR $<$ EPS. At least MINPTS and at most MAXPTS calls to the function F were performed.
- 1 MAXPTS is too small for the specified accuracy EPS. RESULT and RELERR contain the values obtainable for the specified value of MAXPTS.
- 2 IWK is too small for the specified number MAXPTS of function evaluations. RESULT and RELERR contain the values obtainable for the specified value of IRK.
- 3 N $<$ 2, or N $>$ 15, or MINPTS $>$ MAXPTS, or MAXPTS $<$ 2**N $+$ 2N(N $+$ 1) $+$ 1. RESULT and RELERR are set equal to zero.

The user-supplied FUNCTION subprogram F should be of the form

```
FUNCTION F(N,X)
DIMENSION X(*)
...
F =f(X(1),...,X(N)).
RETURN
END
```

where X and F are of type t.

Method:

An integration rule of degree seven is used together with a certain strategy of subdivision. For a more detailed description of the method see **References**.

Error handling:

See description of argument IFAIL.

Notes:

1. Multi-dimensional integration is time-consuming. For each rectangular subregion, the routine requires $2^n + 2n^2 + 2n + 1$ function evaluations. Careful programming of the integrand might result in substantial saving of time.
2. Numerical integration usually works best for smooth functions. Some analysis or suitable transformations of the integral prior to numerical work may contribute to numerical efficiency.

Source:

This subroutine is an adapted version of Fortran program ADAPT published in Ref. 1.

References:

1. A.C. Genz and A.A. Malik, Remarks on algorithm 006: An adaptive algorithm for numerical integration over an N-dimensional rectangular region, J. Comput. Appl. Math. **6** (1980) 295–302.
2. A. van Doren and L. de Ridder, An adaptive algorithm for numerical integration over an n-dimensional cube, J. Comput. Appl. Math. **2** (1976) 207–217.

A copy of the text part of the References is available.

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