CELINT | **CERN Program Library** | **C348**

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Language : Fortran | **Revised:**

Elliptic Integral for Complex Argument

Function subprograms `CELINT` and `WELINT` calculate, for complex argument $z = x + iy$ and real complementary modulus k' a general elliptic integral of the second kind:

$$\mathbf{F}(z, k', a, b) \;=\; \int_0^z \frac{a + b\zeta^2}{(1+\zeta^2)\sqrt{(1+\zeta^2)(1+k'^2\zeta^2)}}\, d\zeta \qquad (k'^2 \geq 0,\ \mathrm{Re}(z) \geq 0),$$

which contains the elliptic integrals of the first and second kind as special cases:

$$\begin{aligned} \mathbf{F}_1(z, k') &= \int_0^z \frac{d\zeta}{\sqrt{(1+\zeta^2)(1+k'^2\zeta^2)}} &&= \mathbf{F}(z, k', 1, 1), \\ \mathbf{F}_2(z, k') &= \int_0^z \frac{d\zeta}{(1+\zeta)^2}\sqrt{\frac{1+k'^2\zeta^2}{1+\zeta^2}} &&= \mathbf{F}(z, k', 1, k'^2). \end{aligned}$$

The double-precision version `WELINT` is available only on computers which support a `COMPLEX*16` Fortran data type.

Structure:

`FUNCTION` subprograms
User Entry Names: `CELINT`, `WELINT`
Files Referenced: `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035)

Usage:

In any arithmetic expression, with $\texttt{AKP} = k'$,

`CELINT(Z,AKP,A,B)` or `WELINT(Z,AKP,A,B)` has the value $\mathbf{F}(\texttt{Z}, k', \texttt{A}, \texttt{B})$,

where `CELINT` is of type `COMPLEX`, `WELINT` is of type `COMPLEX*16`, `Z` is of the same type as the function name, and `AKP`, `A`, `B` are of type `REAL` for `CELINT` and of type `DOUBLE PRECISION` for `WELINT`.

Method:

The evaluation of **F** is based on the Gauss transformation. For details, in particular for the conformal mapping provided by **F**, see Ref. 1.

Accuracy:

`CELINT` (except on CDC and Cray computers) has full single-precision accuracy. For most values of the arguments, `WELINT` (and `CELINT` on CDC and Cray computers) has an accuracy of approximately one significant digit less than the machine precision.

Error handling:

Error `C348.1`: Re $\texttt{Z} < 0$. The function value is set equal to zero, and a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

Notes:

For other forms of the elliptic integrals see the write-up for `RELI1` (C346).

Source:

The subprogram is based on the Algol60 procedure *elco2* given in Ref. 1.

References:

1. R. Bulirsch, Numerical calculation of elliptic integrals and elliptic functions, Numer. Math. **7** (1965) 78–90.

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