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Zeros of Bessel Functions J and Y

Subroutine subprograms RBZEJY and DBZEJY calculate, for real order $a \geq 0$, the first $N > 0$ zeros

$$j_{a,n},\ y_{a,n},\ j'_{a,n},\ y'_{a,n} \qquad (n = 1, 2, \ldots, N)$$

of the Bessel functions $J_a(x)$, $Y_a(x)$, $J'_a(x)$, $Y'_a(x)$, respectively. The prime denotes the derivative of the function with respect to x.

On CDC and Cray computers, the double-precision version DBZEJY is not available.

Structure:

SUBROUTINE subprograms
User Entry Names: RBZEJY, DBZEJY
Obsolete User Entry Names: BZEJY $\equiv$ RBZEJY
Files Referenced: Unit 6
External References: MTLMTR (N002), ABEND (Z035)

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

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CALL tBZEJY(A,N,MODE,REL,X)
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A (type according to t) Order a.

N (INTEGER) Number N of zeros wanted.

MODE (INTEGER) defines the function for which the zeros are to be calculated:
- 1 zeros of $J_a(x)$,
- 2 zeros of $Y_a(x)$,
- 3 zeros of $J'_a(x)$,
- 4 zeros of $Y'_a(x)$.

REL (type according to t) The requested relative accuracy.

X (type according to t) One-dimensional array of length N at least. On exit, X(n), (n = $1, 2, \ldots,$ N) contains the first N positive (in the case A = 0 and MODE = 3, non-negative) zeros of the function defined by MODE.

Method:

Initial approximations to the zeros are computed from asymptotic expansions. These values are improved by higher-order Newton iteration making use of the differential equation for the Bessel functions. (For details see Ref. 1).

Error handling:

Error C345.1: A < 0. A message is written on Unit 6, unless subroutine MTLSET (N002) has been called. The contents of X is left unchanged. N $\leq$ 0 acts as do nothing.

Source:

The subroutine is based on Algol procedures published in the References.

References:

1. N.M. Temme, An algorithm with Algol60 program for the computation of the zeros of ordinary Bessel functions and those of their derivatives, J. Comput. Phys. **32** (1979) 270–279.

2. N.M. Temme, On the numerical evaluation of the ordinary Bessel function of the second kind, J. Comput. Phys. **21** (1976) 343–350.

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