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Bessel Functions J and I with Positive Argument and Non-Integer Order

Subroutine subprograms `BSJA`, `BSIA`, `DBSJA`, `DBSIA` and `QBSJA`, `QBSIA` calculate the sequences of Bessel functions

$$J_{a+n}(x),\ J_{a-n}(x),\ I_{a+n}(x) \text{ or } I_{a-n}(x),$$

for real argument $x > 0$, $0 \le a < 1$, and $n = 0, 1, 2, \ldots, N$.

The quadruple-precision versions `QBSJA` and `QBSIA` are available only on computers which support a `REAL*16` Fortran data type.

Structure:

`SUBROUTINE` subprograms
User Entry Names: `BSJA`, `BSIA`, `DBSJA`, `DBSIA`, `QBSJA`, `QBSIA`
Files Referenced: `Unit 6`
External References: `GAMMA` (C302), `DGAMMA` (C302), `QGAMMA` (C302), `MTLMTR` (N002), `ABEND` (Z035)

Usage:

Single-precision version:

```
CALL BSJA(X,A,NL,ND,B)      or      CALL BSIA(X,A,NL,ND,B)
```

`X` (`REAL`) Argument x.

`A` (`REAL`) Order a of the first Bessel function in the computed sequence.

`NL` (`INTEGER`) Specifies the order $a + \mathtt{NL}$ of the last Bessel function in the computed sequence. It is permissible for `NL` to be negative.

`ND` (`INTEGER`) Requested number of correct significant decimal digits.

`B` (`REAL`) One-dimensional array with dimension (`0:d`) where $\mathtt{d} \ge |\mathtt{NL}|$.
On exit, `B(n)`, ($\mathtt{n} = 0, 1, 2, \ldots, |\mathtt{NL}|$), contains $J_{a+\mathtt{n}}(\mathtt{X})$, $J_{a-\mathtt{n}}(\mathtt{X})$, $I_{a+\mathtt{n}}(\mathtt{X})$ or $I_{a-\mathtt{n}}(\mathtt{X})$, the plus sign of the subscript being taken if $\mathtt{NL} \ge 0$, the minus sign if $\mathtt{NL} < 0$.

Double-precision version:

```
CALL DBSJA(X,A,NL,ND,B)      or      CALL DBSIA(X,A,NL,ND,B)
```

where `X`, `A` and `B` are of type `DOUBLE PRECISION`.

Quadruple-precision version:

```
CALL QBSJA(X,A,NL,ND,B)      or      CALL QBSIA(X,A,NL,ND,B)
```

where `X`, `A` and `B` are of type `REAL*16`.

Method:

For $\mathtt{NL} \ge 0$, the method of computation is a variant of Miller's backwards recurrence algorithm (see Ref. 1). The requested accuracy is obtained, when possible, by a judicious choice of the initial value of the recursion index, together with at least one repetition of the recursion with this index increased by 5. For $\mathtt{NL} < 0$, only the first two functions in the sequence are computed in this way. The remaining functions are computed by the standard Bessel function recurrence relation.

Restrictions:

X > 0, $0 \leq$ A < 1, $|$NL$| \leq 100$.

Accuracy:

If X is close to a zero of one of the functions $J_{a+n}(x)$, the accuracy of that particular function will be less than ND significant digits. There may also be a loss of accuracy in any of the computed functions if A is close to 0 or 1, and in other special situations.

Error handling:

Error C343.1: X ≤ 0.
Error C343.2: A < 0 or A ≥ 1.
Error C343.3: $|$NL$| > 100$.
Error C343.4: Difficulties of convergence. Try smaller $|$ND$|$.
In all cases, a message is written on Unit 6, unless subroutine MTLSET (N002) has been called. If Error 1 to 3 occurs, the initial contents of array B is left unchanged. If the requested accuracy cannot be obtained after the initial recursion index has been increased fifty times (Error 4), the contents of array B is undefined.

Source:

The subprogram is based on Algol procedures described in Ref. 1.

References:

1. W. Gautschi, Algorithm 236, Bessel functions of the first kind, Collected Algorithms from CACM (1972)

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