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Submitter :
Language : Fortran

Library: MATHLIB
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Conical Functions of the First Kind

Function subprograms RFCONC and DFCONC calculate the (real valued) conical function of the first kind

$$P^m_{-\frac{1}{2}+i\tau}(x)$$

for real $x > -1$, $\tau \geq 0$, and $m = 0, 1$, where $P^m_\nu(x)$ is the Legendre (or spherical) function of the first kind and $i = \sqrt{-1}$.
On CDC and Cray computers, the double-precision version DFCONC is not available.

Structure:

FUNCTION subprograms
User Entry Names: RFCONC, DFCONC
Obsolete User Entry Names: FCONC $\equiv$ RFCONC
Files Referenced: Unit 6
External References: CGAMMA (C305), WGAMMA (C305), CLGAMA (C306), WLGAMA (C306), BESJ0 (C312), DBESJ0 (C312), BESJ1 (C312), DBESJ1 (C312), BESI0 (C313), DBESI0 (C313), BESI1 (C313), DBESI1 (C313), RELIKC (C347), DELIKC (C347), RELIEC (C347), DELIEC (C347), MTLMTR (N002), ABEND (Z035)

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

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tFCONC(X,TAU,M)}
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has, in any arithmetic expression, the value $P^{\mathtt{M}}_{\frac{1}{2}+i*\mathtt{TAU}}(\mathtt{X})$.

X (type according to t) Variable x.

TAU (type according to t) The imaginary part of the index, τ.

M (INTEGER) Order m. (M = 0 or M = 1).

Method:

Either (i) series expansions based on the Gaussian hypergeometric function and evaluated by direct summation or from rational approximations, or (ii) asymptotic expressions in terms of Bessel functions. For $\tau = 0$ the conical functions (for $m = 0, 1$) can be expressed in terms of complete elliptic integrals. For details see Ref. 1.

Restrictions:

X ≥ -1, TAU $\geq$ 0, M = 0 or 1.

Accuracy:

RFCONC (except on CDC and Cray computers) has full single-precision accuracy. For most values of the argument X, DFCONC (and RFCONC on CDC and Cray computers), an accuracy of not less than 10 significant digits is usually obtained. If x and τ are not too large the accuracy increases to about 12-13 significant digits.

Error handling:

Error C331.1: $\mathtt{X} < -1$ or $\mathtt{TAU} < 0$ or $\mathtt{M} \neq 0$ and $\mathtt{M} \neq 1$.
Error C331.2: Problems of convergence for a hypergeometric function.
In both cases, the function value is set equal to zero, and a message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

Notes:

This program is an (only formally) modified version of the CPC Program Library Package FCONIC (see Ref. 1).

References:

1. K.S. Kölbig, A program for computing the conical functions of the first kind $P^m_{1/2+i\tau}(x)$ for $m = 0$ and $m = 1$, Computer Phys. Comm. **23** (1981) 51–61.
2. M.I. Zhurina and L.N. Karmazina, Tables and formulae for the spherical functions $P^m_{1/2+i\tau}(z)$, (Pergamon Press, Oxford 1966).

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