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Submitter :
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Nielsen's Generalized Polylogarithm

Function subprograms `CGPLG` and `WGPLG` calculate the complex-valued generalized polylogarithm function

$$S_{n,m}(x) \;=\; \frac{(-1)^{n+m-1}}{(n-1)!\,m!}\int_0^1 t^{-1}\ln^{n-1}t\,\ln^m(1-xt)\,dt \qquad (*)$$

for real arguments x and integer n and m satisfying $1 \le n \le 4,\ 1 \le m \le 4,\ n+m \le 5$; i.e., one of the functions $S_{1,1}$, $S_{1,2}$, $S_{2,1}$, $S_{1,3}$, $S_{2,2}$, $S_{3,1}$, $S_{1,4}$, $S_{2,3}$, $S_{3,2}$, $S_{4,1}$. If $x \le 1$, $S_{n,m}(x)$ is real, and the imaginary part is set equal to zero.

The double-precision version `WGPLG` is available only on computers which support a `COMPLEX*16` Fortran data type.

Structure:

`FUNCTION` subprograms
User Entry Names: `CGPLG`, `WGPLG`
Files Referenced: `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035)

Usage:

In any arithmetic expression,

`CGPLG(N,M,X)` or `WGPLG(N,M,X)` has the value $S_{\mathtt{N},\mathtt{M}}(\mathtt{X})$,

where `CGPLG` is of type `COMPLEX`, `WGPLG` is of type `COMPLEX*16`, `X` is of type `REAL` for `CGPLG` and of type `DOUBLE PRECISION` for `WGPLG`, and where `N` and `M` are of type `INTEGER`.

Method:

The method is described in Ref. 1. Note that the imaginary part of the function defined as $S_{n,m}(x)$ in Ref. 1 has the opposite sign to the imaginary part of the function defined by (*). See Ref. 2.

Accuracy:

`CGPLG` (except on CDC and Cray computers) has full single-precision accuracy. For most values of the argument `X`, `WGPLG` (and `CGPLG` on CDC and Cray computers) has an accuracy of approximately two significant digits less than the machine precision. The loss of accuracy is greater when `X` is very close to `1`.

Error handling:

Error `C321.1`: $\mathtt{N},\mathtt{M} < 1$ or $\mathtt{N},\mathtt{M} > 4$ or $\mathtt{N}+\mathtt{M} > 5$. The function value is set equal to zero, and a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

References:

1. K.S. Kölbig, J.A. Mignaco and E. Remiddi, On Nielsen's generalized polylogarithms and their numerical calculation, BIT **10** (1970) 38–71.
2. K.S. Kölbig, Nielsen's generalized polylogarithms, SIAM J. Math. Anal. **17** (1986) 1232–1258.

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