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# Jacobian Elliptic Functions sn, cn, dn for Complex Argument

Function subprograms `CELFUN` and `WELFUN` calculate, for complex argument $z$ and real modulus $k$, the Jacobian elliptic functions $\mathrm{sn}(z,k)$, $\mathrm{cn}(z,k)$ and $\mathrm{dn}(z,k)$. The function $\mathrm{sn}(z,k)$ is the inverse of the elliptic integral of the first kind and is defined implicitly by

$$z = \int_0^{\mathrm{sn}(z,k)} \frac{dw}{\sqrt{(1-w^2)(1-k^2w^2)}} \qquad (k^2 \le 1).$$

The functions $\mathrm{cn}(z,k)$ and $\mathrm{dn}(z,k)$ are defined by

$$\mathrm{sn}^2(z,k) + \mathrm{cn}^2(z,k) = 1, \quad k^2\mathrm{sn}^2(z,k) + \mathrm{dn}^2(z,k) = 1, \quad \mathrm{cn}(0,k) = \mathrm{dn}(0,k) = 1.$$

For $k = 0$ and $k^2 = 1$ these functions are elementary:

$$\mathrm{sn}(z,0) = \sin z, \quad \mathrm{cn}(z,0) = \cos z, \quad \mathrm{dn}(z,0) = 1,$$

$$\mathrm{sn}(z,\pm1) = \tanh z, \quad \mathrm{cn}(z,\pm1) = \mathrm{dn}(z,\pm1) = \mathrm{sech}\, z.$$

Note that the Jacobian elliptic functions are doubly-periodic in the $z$-plane. For details see the relevant treatises or handbooks.

The double-precision version `WELFUN` is available only on computers which support a `COMPLEX*16` Fortran data type.

**Structure:**

`SUBROUTINE` subprograms
User Entry Names: `CELFUN`, `WELFUN`
External References: `MTLMTR` (N002), `ABEND` (Z035)

**Usage:**

For `t` = `C` (type `COMPLEX`), `t` = `W` (type `COMPLEX*16`),

```
CALL tELFUN(Z,AK2,SN,CN,DN)
```

`Z` (type according to `t`) The argument $z$.

`AK2` (`REAL` for `t` = `C`, `DOUBLE PRECISION` for `t` = `W`) The value of $k^2$ (the square of the modulus).

`SN` (type according to `t`) On exit, `SN` = $\mathrm{sn}(Z,k)$.

`CN` (type according to `t`) On exit, `CN` = $\mathrm{cn}(Z,k)$.

`DN` (type according to `t`) On exit, `DN` = $\mathrm{dn}(Z,k)$.

**Method:**

The Jacobian elliptic functions with complex argument $z = x + iy$ are computed from their representations in terms of Jacobian elliptic functions with real arguments $x$ or $y$ (Ref. 1, formula 125.01). See also the Short Write-up for `ELFUN` (C318).

**Accuracy:**

CELFUN (except on CDC and Cray computers) has full single-precision accuracy. For most values of the arguments, WELFUN (and CELFUN on CDC and Cray computers) has an accuracy of approximately two significant digits less than the machine precision.

**Restrictions:**

$|\mathrm{Re}\, z| \leq 3\mathrm{K}(k)$, $|\mathrm{Im}\, z| \leq 3\mathrm{K}(k')$ where $k' = \sqrt{1-k^2}$ is the complementary modulus, and $\mathrm{K}(x)$ is the complete elliptic integral of the first kind. (See entries RELIKC and DELIKC in RELI1C (C347)).

**Error handling:**

Error C320.1: $|\mathtt{AK2}| > 1$. The function value is set equal to zero, and a message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

**References:**


1. P.F. Byrd and M.D. Friedman, Handbook of elliptic integrals for engineers and scientists, 2nd ed., Springer-Verlag Berlin (1971).


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