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Modified Bessel Functions I and K of Orders Zero and One

Function subprograms BESI0, BESI1, BESK0, BESK1 and DBESI0, DBESI1, DBESK0, DBESK1 calculate the modified Bessel functions

$$I_0(x),\ I_1(x),\ K_0(x),\ K_1(x)$$

for real arguments x, where $x > 0$ for $K_0(x)$ and $K_1(x)$.

On CDC and Cray computers, the double-precision versions DBESI0 etc. are not available.

Structure:

FUNCTION subprograms
User Entry Names: BESI0, BESI1, BESK0, BESK1, EBESI0, EBESI1, EBESK0, EBESK1,
DBESI0, DBESI1, DBESK0, DBESK1, DEBSI0, DEBSI1, DEBSK0, DEBSK1
Files Referenced: Unit 6
External References: MTLMTR (N002), ABEND (Z035)

Usage:

In any arithmetic expression,

BESI0(X)	or	DBESI0(X)	has the value	I_0(X),
BESI1(X)	or	DBESI1(X)	has the value	I_1(X),
BESK0(X)	or	DBESK0(X)	has the value	K_0(X),
BESK1(X)	or	DBESK1(X)	has the value	K_1(X),
EBESI0(X)	or	DEBSI0(X)	has the value	$\exp(-\lvert$X$\rvert) * I_0$(X),
EBESI1(X)	or	DEBSI1(X)	has the value	$\exp(-\lvert$X$\rvert) * I_1$(X),
EBESK0(X)	or	DEBSK0(X)	has the value	$\exp(\lvert$X$\rvert) * K_0$(X),
EBESK1(X)	or	DEBSK1(X)	has the value	$\exp(\lvert$X$\rvert) * K_1$(X),

where BESI0 etc. are of type REAL, DBESI0 etc. are of type DOUBLE PRECISION, and X has the same type as the function name.

Method:

Approximation by rational functions (I for $|x| < 8$, K for $1 \leq x \leq 5$), by an algorithm based on power series (K for $0 < x < 1$), or else by truncated Chebyshev series.

Accuracy:

BESI0 etc. (except on CDC and Cray computers) have full single-precision accuracy. For most values of the argument X, DBESI0 etc. (and BESI0 etc. on CDC and Cray computers) have an accuracy of approximately one significant digit less than the machine precision.

Error handling:

Error C313.1: X $\leq$ 0 for $K_0(x)$ or $K_1(x)$. The function value is set equal to zero, and a message is written on Unit 6 unless subroutine MTLSET (N002) has been called.

References:

1. Y.L. Luke, Mathematical functions and their approximations (Academic Press, New York 1975) 329, 331, 363, 366.
2. N.M. Temme, On the numerical evaluation of the modified Bessel function of the third kind, J. Comp. Phys. **19** (1975) 324–337.

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