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Zeros of a Real Polynomial

Subroutine subprogram RMULLZ and DMULLZ compute the zeros of the polynomial

$$P(z) \;=\; a_0 z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n$$

of degree n with real coefficients a_k and $a_0 \neq 0$.
On computers other than CDC or Cray, only the double-precision version DMULLZ is available. On CDC and Cray computers, only the single-precision version RMULLZ is available.

Structure:

SUBROUTINE subprograms
User Entry Names : RMULLZ, DMULLZ
Files Referenced: Unit 6
External References: MTLMTR (N002), ABEND (Z035)

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

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CALL tMULLZ(A,N,MAXIT,Z)
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A (type according to t) One-dimensional array of dimension (0:d), where d $\geq$ N, containing the coefficients a_k, $(k = 0, 1, \ldots, n)$.

N (INTEGER) The degree n.

MAXIT (INTEGER) The maximum number of iterations permitted.

Z (COMPLEX for t = R, COMPLEX*16 for t = D) One-dimensional array of length $\geq$ N. On exit, Z(i) contains an approximation to the zero z_i, listed in roughly decreasing order of $|z_i|$.

Method:

The method of Muller (see Ref. 1) is used. This is based on iterated inverse quadratic interpolation followed by deflation to remove each zero as found.

Accuracy:

For well-conditioned polynomials (i.e. polynomials whose zeros are not unduly sensitive to small errors in the coefficients), the relative error of a computed zero of multiplicity m is of order $10^{-d/m}$ where d is the machine precision expressed in decimal digits. For $m > 1$, the m approximations to the single multiple zero are uniformly distributed on a small circle of radius of order $10^{-d/m}$ around the exact zero. Therefore, if the polynomial is well-conditioned, the true value of the multiple zero will be close to the centre $(z_{k+1} + \ldots + z_{k+m})/m$ of this circle.

Error handling:

Error C202.1: $a_0 = 0$.
Error C202.2: The number of iterations exceeds MAXIT.
In both cases, a message is written on Unit 6, unless subroutine MTLSET (N002) has been called. If the number of iterations exceeds MAXIT, those zeros which have not been found are set to 10^{20}.

Notes:

For difficult cases which lead to too many iterations the following transformations may be applied, singly or together, to obtain a better-conditioned polynomial:

1. Reverse the order of the coefficients to obtain a polynomial whose zeros are z_i^{-1}.
2. If the zeros z_i are clustered, or are too unsymmetrically positioned with respect to the origin, compute by synthetic division (see Ref. 3) the coefficients of the polynomial whose argument is $w = z - \widehat{z}$, where $\widehat{z} = -a_1/(na_0)$ is the arithmetic mean of the zeros. The mean of the zeros of this new polynomial is situated at the origin, which is where the subprogram starts searching. Then, provided $|w_i| < |\widehat{z}|$ for most i, $z_i = w_i + \widehat{z}$ will be more accurate zeros.

References:

1. D.E. Muller, A method for solving algebraic equations using an automatic computer, MTAC (later renamed Math. Comp.) **10** (1956) 208–215.
2. J.W. Daniel, Correcting approximations to multiple roots of polynomials, Numer. Math. **9** (1966) 99–102.
3. F.B. Hildebrand, Introduction to numerical analysis, McGraw-Hill, New York (1956), Section 10.9.

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